UNSTEADY CONJUGATE HEAT EXCHANGE BETWEEN A SEMIINFINITE SURFACE AND A STREAM OF COMPRESSIBLE FLUID FLOWING OVER IT

III. NUMERICAL CALCULATION

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The paper presents the results of a numerical calculation of the function $\Omega_n(\tau^*)$ through which one expresses the solution of the problem of conjugate unsteady heat exchange between a plate containing heat sources and a gas stream flowing over one side.

In [1, 2] the solution of the problem of conjugate unsteady heat exchange between a semiinfinite plate (with heat sources of the type $R(z, \tau) = R_n(\tau)z^n$) and a gas stream flowing over one side is expressed through the function $\Omega_n(\tau)$. In [2] it is shown that $\Omega_n(\tau)$ is represented by a power-law series. The results of a numerical calculation of the function $\Omega_n(\tau)$ are presented in the present work.

The mathematical formulation of the problem discussed in [1, 2] has the form (in dimensionless form)

$$\frac{\partial^2\theta}{\partial\eta^2} + \Pr f \frac{\partial\theta}{\partial\eta} = \Pr f' \xi \frac{\partial\theta}{\partial\xi} + B\xi^2 \frac{\partial\theta}{\partial\tau}, \tag{1}$$

$$\frac{\partial\theta|_{\eta=0}}{\partial\tau} = \frac{\partial^2\theta|_{\eta=0}}{\partial z^2} + \frac{M}{\sqrt{z}}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0} + R(z, \tau), \tag{2}$$

$$\left(\frac{\partial\theta|_{\eta=0}}{\partial z}\right)_{z=0} = \left(\frac{\partial\theta|_{\eta=0}}{\partial z}\right)_{z\to\infty} = 0, \tag{3}$$

$$\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi=0} = 0, \quad \theta|_{\eta\to\infty} = 0, \quad \theta|_{\tau=0} = 0. \tag{4}$$

Here

$$B = \frac{2}{\mathrm{Re}}\frac{a_s}{Ca_\infty}, \quad M = \frac{\lambda_\infty}{\lambda_s}\left(\frac{\mathrm{Re}}{2}\right)^{1/2}\frac{L}{H}\sqrt{C}.$$

The meaning of the other quantities which figure in the expressions presented above is revealed in [1].

Before starting on the numerical calculation, let us transform the problem (1)-(4): $\xi, \tau, z \to \xi^* = M^{1/3}\xi$, $\tau^* = M^{4/3}\tau$, $z^* = M^{2/3}z$. Then (1)-(4) takes the form

$$\frac{\partial^2\theta}{\partial\eta^2} + \Pr f \frac{\partial\theta}{\partial\eta} = \Pr f' \xi^2 \frac{\partial\theta}{\partial\xi^*} + B^*\xi^{*2} \frac{\partial\theta}{\partial\tau^*}, \tag{5}$$

$$\frac{\partial\theta|_{\eta=0}}{\partial\tau^*} = \frac{\partial^2\theta|_{\eta=0}}{\partial z^{*2}} + \frac{1}{\sqrt{z^*}}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0} + \frac{1}{M^{4/3}} R\left(\frac{z^*}{M^{2/3}}, \frac{\tau^*}{M^{4/3}}\right), \tag{6}$$

$$\left(\frac{\partial\theta|_{\eta=0}}{\partial z^*}\right)_{z^*=0} = \left(\frac{\partial\theta|_{\eta=0}}{\partial z^*}\right)_{z^*\to\infty} = 0, \tag{7}$$

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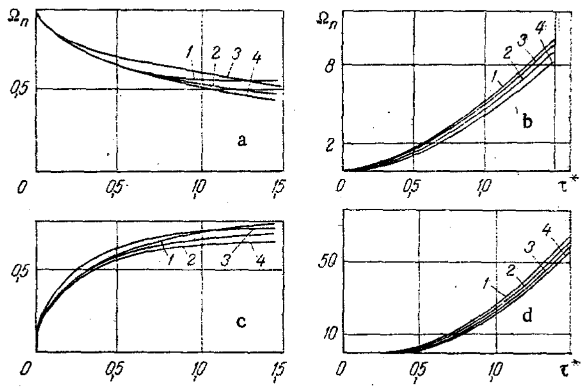

Fig. 1. Graphs of functions $\Omega_n(\tau^*)$ (n = 0–5) with Pr = 1 for different approximations N = 1 (1); 4 (2); 7 (3); 10 (4); a) N = 0; b) 4; c) 1; d) 5.

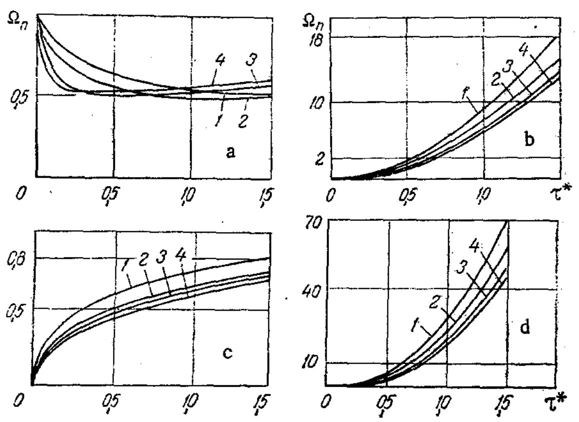

Fig. 2. Graphs of functions $\Omega_n(\tau^*)$ (n = 0–5) in the first approximation for Pr = 1 (1); 7.03 (2); 50 (3); 100 (4) with N = 1: a) n = 0; b) 4; c) 1; d) 5.

$$\left(\frac{\partial\theta}{\partial\xi}\right)_{\xi^*=0}=0,\quad \theta|_{\eta\to\infty}=0,\quad \theta|_{\tau^*=0}=0, \tag{8}$$

where $B^* = BM^{2/3}$.

Precisely the problem (5)–(8) is analyzed in the present report, since for it the function $\Omega_n(\tau^*)$ does not contain the quantity M explicitly.

A comparison of Eqs. (1)–(4) and (5)–(8) shows that the solution of problem (5)–(8) is obtained from the solution found in [1, 2] for problem (1)–(4) by the substitution

$$\xi,\ z,\ \tau,\ M,\ B \to \xi^*,\ z^*,\ \tau^*,\ 1,\ B^*.$$

The solution of problem (5)–(8) can be calculated by Eqs. (43), (50), (51), (89)–(92), and (115)–(118) from [1, 2] starting from an assigned heat source. Only knowledge of the function $\Omega_n(\tau^*)$, which does not depend on the source, is required for these calculations.

The results of a calculation of the function $\Omega_n(\tau^*)$ in the quasisteady case (i.e., a quasisteady boundary layer and an unsteady substance) are presented in this report. Approximate equations for calculating $\Omega_n(\tau^*)$ in the N-th approximation are obtained in Sec. 10 of [2]. The calculations were carried out in the approximations N = 3m + 1, since one can show that precisely these are the best approximations.

Graphs of the function $\Omega_n(\tau^*)$ for different approximations N are presented in Fig. 1. It is seen from the figure that the first approximation already represents the function $\Omega_n(\tau^*)$ rather well.

The results of the calculation of the functions $\Omega_n(r^*)$ in the first approximation for Pr = 1, 7.03, 50, and 100 are shown in Fig. 2. The weak dependence of the functions $\Omega_n(r^*)$ on the Prandtl number is seen from the figure.

NOTATION

θ, dimensionless temperature; $a_s(a_\infty)$, coefficient of thermal diffusivity of plate (of onflowing gas); $\lambda_s(\lambda_\infty)$, coefficient of thermal conductivity of plate (of onflowing gas); L(H), length (thickness) of plate; C, constant in Chapman–Rubezin law.

LITERATURE CITED

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CONJUGATE PROBLEM OF STEADY HEAT EXCHANGE IN THE LAMINAR FLOW OF AN INCOMPRESSIBLE FLUID IN A FLAT CHANNEL

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The conjugate problem of convective heat exchange in a flat channel is solved by the combined application of the integral Laplace transformation and the Bubnov–Galerkin method.

Considerable attention has recently been paid to problems of conjugate heat exchange [1].

Results of numerical calculations of the conjugate problem of steady heat exchange are presented in [2, 3], while an exact solution of the very general problem of conjugate heat exchange with boundary conditions of the first kind at the surface of the pipe wall is obtained analytically in [4]. However, practical use of the solutions obtained is made difficult by the complicated and cumbersome functional dependences.

The combined application of the integral Laplace transformation and the Bubnov–Galerkin method [5] allows one to obtain an approximate solution of the conjugate problem which is suitable for direct calculations.

Let us make the following assumptions: the flow of the fluid and the process of heat exchange are steady; the heat-transfer agent is viscous and incompressible; the mode of flow is laminar; the temperature of the heat-transfer agent is constant in the entrance section of the channel; the temperature of the outer surface of the channel walls is an arbitrary function of the longitudinal coordinate; the curvature of the temperature distribution in the fluid in the longitudinal direction can be neglected in comparison with the curvature in the transverse direction – this assumption is evaluated in [6]; the temperature field is axisymmetric. With allowance for these assumptions the energy equation for the fluid in dimensionless variables has the form

$$\frac{3}{2}(1-\xi^2)\frac{\partial\Theta_1(\xi, X)}{\partial X} = \frac{\partial^2\Theta_1(\xi, X)}{\partial\xi^2} \quad (0<\xi<1,\ 0<X<\infty). \tag{1}$$

We assume that the channel wall is made of an anisotropic material (here the coordinate system coincides with the principal coordinate system), and then the heat-conduction equation for the wall is written in the form

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